

**Goal:** Find an inverse of a function from a graph or equation.



Questions

**Warm Up:** Indicate how you could undo each operation or composite of functions.

1. Turning left and walking 30 yards, then turning around and walking 15 yards
2. Multiplying a number by  $1\frac{2}{3}$
3. Adding 40 to a number,. Then dividing the result by one-half
4. Taking the fifth power of a positive number, then taking it positive square root.

If the volume of a cube can be found using the formula  $V = e^3$ , where  $e$  is an edge length and you knew that a cube was  $250 \text{ cm}^3$ , what would  $e$  be?

### Finding the Inverse of a Function

A function can be considered as a set of ordered pairs in which each first element is paired \_\_\_\_\_. If you switch coordinates in the pairs, the resulting set of ordered pairs is called the \_\_\_\_\_.

Example 1: Let  $f = \{(-3,-5),(-2,0),(-1,3),(0,4),(1,3),(2,0),(3,-5)\}$ .

Describe the inverse of  $f$ . Is the inverse a function?

### Practice 1:

Let  $h = \{(1, 1), (2, 4), (3, 9), (4, 16)\}$ .

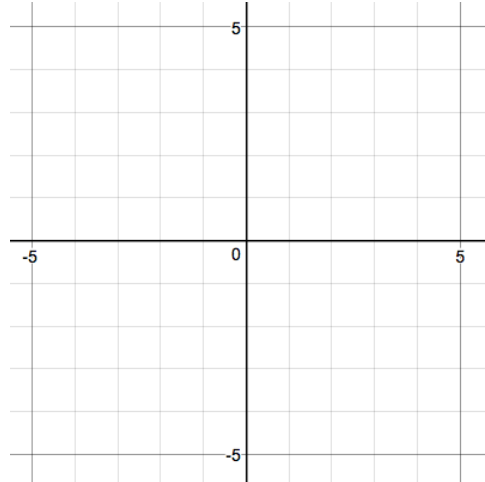
- a. Describe the inverse of  $h$ .
- b. Is the inverse a function?
- c. Describe  $h$  and its inverse in words.

Questions

If you are given a function, to find the inverse, all you need to do is \_\_\_\_\_ . If the inverse is a function, then \_\_\_\_\_ .

Example 2:

- Give an equation for the inverse of the function described by  $y = -x^2 + 4$  .
- Sketch a graph of  $y = -x^2 + 4$  and the inverse on the same set of axes.
- Is the inverse a function?



What do you notice about the graphs of the equations?

What do you notice about the tables at the right?

function $y = -x^2 + 4$		inverse $x = -y^2 + 4$	
x	y	x	y
-2	0	0	-2
-1	3	3	-1
0	4	4	0
1	3	3	1
2	0	0	2
3	-5	-5	3
4	-12	-12	4

Practice 2:

- Describe the graph of the function  $y = 2(x + 5)^2 - 1$  .
- Give an equation for the inverse of the function  $y = 2(x + 5)^2 - 1$  .
- Based on your answer to Part a, describe the graph of the inverse of the function. Is the inverse a function?

## Questions

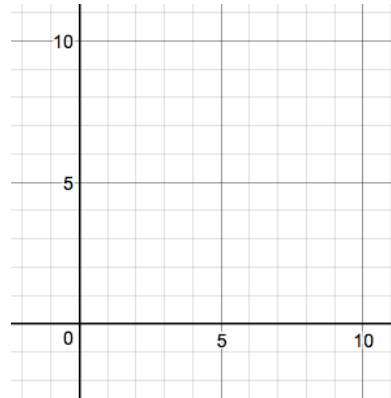
Activity:

1. If  $a(x) = \frac{1}{x-3} + 4$ , describe what the function does to  $x$  according to the order of operations.
2. Describe the process to “undo”  $a(x)$ . Call this function  $b$  and write a formula for  $b(x)$ .
3. Evaluate  $(a \circ b)(8)$ . What do you notice?

Example 3: Consider the function  $f$  with

$$f(x) = \frac{1}{x-3} + 4.$$

- a. Give an equation for the inverse of  $f$ .
- b. Graph  $f$  and its inverse on the same set of axes.

Practice 3: Consider the function  $f$  with  $h(x) = \frac{1}{x+8} - 1$ .

- a. Give an equation for the inverse of  $h$ .
- b. Is the inverse of  $h$  a function?

## Questions

Examples 2 and 3 show us something true of about inverses: \_\_\_\_\_ . When the inverse of a function is a \_\_\_\_\_, then it is denoted by the symbol  $f^{-1}$ , read “ $f$  inverse.” BEWARE: this does not mean \_\_\_\_\_.

**Inverse Functions and Compositions of Functions**

Finding the inverse of a function requires you to switch the  $x$ - and  $y$ -coordinates. Therefore switching the domain and range of the \_\_\_\_\_, gives us the domain od range of the \_\_\_\_\_.

**Inverses of Functions Theorem**

Given any two functions  $f$  and  $g$ ,  $f$  and  $g$  are inverse functions if and only if  $f(g(x)) = x$  for all  $x$  in the domain of  $g$ , and  $g(f(x)) = x$  for all  $x$  in the domain of  $f$ .

Example 4: Use the inverses of Functions Theorem to determine whether  $f$  and  $g$ , defined by  $f(x) = \sqrt{2x-4}$   $g(x) = \frac{x^2+4}{2}$ , are inverses. Verify by graphing.

Practice 4: Use the inverses of Functions Theorem to determine whether  $f(x) = \frac{1}{x+5} - 3$  and  $g(x) = \frac{1}{x-5} + 3$ .

## Summary: